5.4 Notes – Logarithmic Models

One of the most common applications of logarithms is in creating measurement scales such as:

→ Decibels
→ pH Scale
→ Richter Scale

The **decibel**, named after the inventor of the telephone, Alexander Graham Bell (1847-1922), is defined as: $D = 10\log\frac{I}{I_0}$ *where* **D** = Decibel Level (of sound)

I = intensity (of sound) *(watts per square meter)*

I_0 = 10^{-12} *(the least audible sound that an average healthy person can hear)* (watts/m²)

Ex 1: The average sound intensity of a whisper is 5.2 x 10 $^{-10}$ watts per square meter. The average sound intensity of heavy traffic is 8.5 x 10 $^{-4}$ watts per square meter.

a. Determine the decibel level of each of these sounds.

whisper: $D = 10\log\left(\frac{5.2\times10^{-10}}{10^{-12}}\right)$

$D = 10\log(5.2\times10^{2})$

$D \approx 27.16$ decibels

heavy traffic: $D = 10\log\left(\frac{8.5\times10^{-4}}{10^{-12}}\right)$

$D = 10\log(8.5\times10^{8})$

$D \approx 89.29$ decibels

b. In terms of sound intensity, how many times louder is heavy traffic than a whisper?

$\frac{8.5\times10^{-4}}{5.2\times10^{-10}} \approx 1{,}634{,}615.39$ times louder!

Ex 2: At one Smashing Pumpkins concert, the sound in the front row was measured at 110 decibels. What sound intensity is this and how does it compare to the sound of heavy traffic?

$D = 10\log\left(\frac{I}{I_0}\right)$

$110 = 10\log\left(\frac{I}{10^{-12}}\right)$

$11 = \log\left(\frac{I}{10^{-12}}\right)$

$10^{11} = \frac{I}{10^{-12}}$

$I = .1$

compare: $\frac{\text{pumpkins}}{\text{traffic}} = \frac{.1}{8.5\times10^{-4}} = 117$ times louder.

In 1935 the California seismologist Charles Richter devised a logarithmic scale to measure the destructive power of earthquakes.

The **magnitude**, is defined as: $M = \frac{2}{3}\log\frac{E}{E_0}$

M = Magnitude (of earthquake)

E = energy released *(joules)*

$E_0 = 10^{4.40}$ joules *(the energy released by a very small reference earthquake)*

The 1906 San Francisco earthquake released approximately 5.96×10^{16} joules of energy. Another quake struck the Bay Area just before game 3 of the 1989 World Series, releasing 1.12×10^{15} joules of energy.

a. Find the magnitude of each earthquake on the Richter scale. *Round to the nearest hundredth.*

1906: $M = \frac{2}{3}\log\left(\frac{5.96 \times 10^{16}}{10^{4.4}}\right)$

$M \approx 8.25$

1989: $M = \frac{2}{3}\log\left(\frac{1.12 \times 10^{15}}{10^{4.4}}\right)$

$M \approx 7.1$

b. How many times more energy did the 1906 earthquake release than the one in 1989?

Note: ENERGY (not magnitude :))

$\frac{1906}{1989}$: $\frac{5.96 \times 10^{16}}{1.12 \times 10^{15}} \approx 53.2$ times larger

Using your phone… 

Look up the most intense earthquake recorded in the United States, using the Richter scale.

When and where did it occur? March 28, 1964 – Alaska

What was the magnitude? 9.2

What was the energy released (based on our formula)? 1.58×10^{18} joules

$9.2 = \frac{2}{3}\log\left(\frac{x}{10^{4.4}}\right) \rightarrow 13.8 = \log\left(\frac{x}{10^{4.4}}\right)$

$10^{13.8} = \frac{x}{10^{4.4}}$

Using your phone, look up the most intense earthquake recorded in the World, using the Richter scale.

When and where did it occur? May 22, 1960 - Valdivia Chile

What was the magnitude? 9.5

What was the energy released (based on our formula)? 4.47×10^{18}

$9.5 = \frac{2}{3}\log\left(\frac{x}{10^{4.4}}\right)$

$10^{14.25} = \frac{x}{10^{4.4}}$

$14.25 = \log($ " " $)$

For the One Note….

In this section we will study logarithmic scales that are used to compare the intensity of sounds, the severity of earthquakes, and the brightness of distant stars.

Sound Intensity: The human ear is able to hear sound over a very wide range of intensity. The loudest sound a healthy person can hear without damage to the eardrum has an intensity of 1 trillion times that of of the softest sound a person can hear. If we are to use these intensities as a scale of measuring volume, we would be stuck using numbers from zero all the way to the trillions, which seems cumbersome, in not downright silly. IN the last section, we saw that logarithmic function increase very slowly. We can take advantage of this to create a scale for sound intensity that is much more condensed, and therefore much more manageable.